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The influence of projecting sidewalls on the hydrodynamic performance of wave-energy devices

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The concept of adding a harbour, consisting of two parallel projections to a wave-energy device was first brought to the attention of the wave-energy community at a Symposium in Trondheim, Norway, in June 1982. The proponents of the idea claim that the performance of the device is considerably improved by the addition of the harbour, thereby reducing costs. In this paper two theoretical techniques are described for predicting the performance of the harbour system. First, a relatively simple approximate method using the theory of long thin harbours is described. Secondly, numerical techniques used for rigid-body interaction with waves are adapted to cope with harbour systems with no restrictions on dimensions. It is shown that the simpler approach gives results that agree closely with numerical calculations over a wide range of configurations. Hydrodynamic theory is used to evaluate the performance of the device, assuming that it can absorb energy through a resistive damper. The results are encouraging, demonstrating that the addition of a harbour can be very beneficial and confirming that the concept is worthy of closer scrutiny.

1. Introduction

Although the idea of extracting useful energy from ocean waves is not new (see Stahl 1892) it is only in recent years that concerted efforts have been made to invent devices that will capture the energy both efficiently and cheaply. An up-to-date description of many of these devices is given by Shaw (1982) and Count, Fry & Haskell (1983). Concurrent with engineering development and experimentation, considerable progress has also been made on the theoretical hydrodynamics of these devices, and a fairly exhaustive list of references up to 1980 can be found in the review article by Evans (1981).

Although the theoretical work has been almost exclusively restricted to linear classical water-wave theory, comparison with experiment has been generally satisfactory, as shown in the paper by Count & Jefferys (1980). As more sophisticated devices evolve, it becomes increasingly difficult to develop simple analytical results, and greater use of numerical methods is necessary. Nevertheless, such methods are costly in both time and money, particularly where detailed parametric studies are required. It is always desirable, therefore, to construct good, accurate, simple approximate solutions, wherever possible.

Such a solution is presented here for a novel wave-energy device invented by a

Norwegian group (Ambli *et al.* 1982). Most wave-energy devices are resonant in the sense that their active elements are tuned to respond most at a particular wave frequency. This resonance can be achieved in different ways; for example in the Salter 'duck' device resonance is achieved by balancing the inertia of the rocking 'duck' plus its hydrodynamic added inertia against the hydrostatic restoring forces as its immersed volume changes. Again, for a typical isolated oscillating water column (Moody & Elliott 1982) the dimensions are chosen so as to resonate a trapped mass of water with its own hydrostatic restoring force.

It is the oscillating water column to which the Norwegian idea has been applied, but there is no reason why other types of devices could not be used. The idea is to build thin projecting sidewalls out from a device towards the incoming waves so as to create an additional resonance within the harbour so formed. The simplest analogy is the quarter-wavelength effect in acoustics, whereby large amplification of the motion inside a parallel-sided duct with one end closed is achieved by exciting the open end with a disturbance having a frequency corresponding to a wave four times the length of the duct. In the wave-energy case the incoming wave field is thereby considerably amplified before it reaches the active elements of the device, with a consequent potential increase in efficiency.

The plan of the paper is as follows. In §2 the problem is formulated and a necessary summary of the theory of isolated wave-energy devices presented. A simple approximate technique is then used to derive modified expressions for all hydrodynamic quantities of interest, including the capture-width ratio, defined as the mean power captured per unit wavefront of the incoming waves, for a device modified by the inclusion of projecting sidewalls of arbitrary length. The approximate formulae derived are applied to a simple idealized device and the results compared with a full numerical procedure based on boundary integral methods. Agreement between the two methods is shown to be convincing even for short harbour walls, where the approximate theory can be expected to work least well.

Finally an assessment is made of the effectiveness of adding sidewalls to this idealized wave-energy device, and it is shown that considerable improvement of performance can be achieved, provided that the length of the walls is chosen carefully.

2. Formulation and approximate solution

We fix attention on an idealized device consisting of a rectangular block which extends through the entire water depth as shown in figure 1. The vertical side of the block facing the incoming waves is free to make small simple harmonic horizontal oscillations in response to a monochromatic incident wave. This motion is assumed to be resisted by some linear internal mechanism, such as a pump, so that work is done on the device. It is further assumed that the two vertical faces bounding the energy-absorbing face are extended outwards to meet the incoming waves by the addition of thin sidewalls so as to form a kind of 'harbour', as shown in figure 1.

The aim of the paper is twofold. First to develop and validate a simple theoretical model enabling the performance of such a device to be predicted, and secondly to assess the effect on power absorption by the addition of the projecting sidewalls.

2.1. *Theory of isolated wave-energy devices*

The theory of an isolated rigid body absorbing energy by oscillations involving a single degree of freedom, in either two or three dimensions, has been developed in different ways by a number of authors. For a review see Evans (1981).

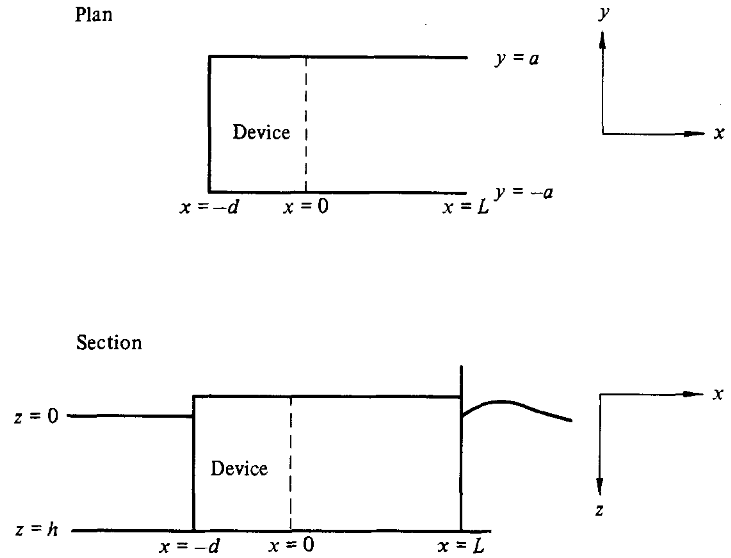

FIGURE 1. The geometry of a harbour system.

A description suited to our purposes is to consider simple harmonic motions at radian frequency ω. With the time dependence $\exp(-\mathrm{i}\omega t)$ removed, the equation of motion of the device can be written as

$$X_e + X = ZU, \tag{2.1}$$

where

$$X_e = -\Lambda U \tag{2.2}$$

is the force opposing the motion of the active front face, taken to be proportional to the (complex) velocity U of that face. The horizontal wave-exciting force on the front face is X and the complex impedance is

$$Z = B - \mathrm{i}\omega\left(I + M - \frac{C}{\omega^2}\right), \tag{2.3}$$

where I is the inertia (here just the mass) of the front face, M and B respectively are the frequency-dependent added-mass and radiation-damping coefficients, and C is a constant form corresponding to the linearized hydrostatic restoring force (here $C = 0$).

The mean power P *absorbed* is minus the mean rate of working of the applied force X_e, so that

$$P = -\mathrm{Re}\,\tfrac{1}{2}\bar{X}_e U = \tfrac{1}{4}(\Lambda + \bar{\Lambda})\,|U|^2 \quad \text{(from (2.2))}$$

$$= \frac{1}{8}\frac{|X|^2}{B}\left\{1 - \frac{|\Lambda - \bar{Z}|^2}{|\Lambda + Z|^2}\right\} \quad \text{(from (2.1)–(2.3))}, \tag{2.4}$$

where a bar denotes the complex conjugate. Clearly

$$P_{max} = \frac{|X|^2}{8B} \quad \text{if} \quad \Lambda = \bar{Z}, \tag{2.5}$$

although generally Λ is real and positive, in which case

$$P_{max} = \frac{|X|^2}{4B(|Z| + B)} \quad \text{when} \quad \Lambda = |Z|. \tag{2.6}$$

Equation (2.4) applies to any device, including the present idealized one, with any length of projecting sidewalls. In particular it holds when the sidewall length L is either zero or infinite. The latter case would correspond to the rectangular block spanning a narrow semi-infinite wave tank and operating like a piston wavemaker in the presence of an incident wavetrain. (It will be assumed throughout this paper that the incident wavelength is greater than twice the width of the front face, so that, even in obliquely incident waves, only the fundamental, with crests perpendicular to the sidewalls, is incident upon the active front face.) The wavemaker comparison is useful, since it is easily seen, by considering the time-reversal of the forced motion of the wavemaker in the *absence* of an incident wave, that it is always possible in theory to absorb *all* the energy in an incoming wavetrain. In practice, of course, this will not be possible at all frequencies unless A is allowed to vary in order to satisfy $A = \bar{Z}$.

The conclusion of total absorption is confirmed by substitution in (2.5) of the relation

$$|X|^2 = 8BP_{\mathrm{W}} \tag{2.7}$$

connecting the exciting force with the mean power P_{W} incident on the device. Equation (2.7) holds for cylindrical sections completely spanning the narrow wave tank making simple harmonic oscillations in regular waves of amplitude A, and can be deduced from more general results derived in Newman (1976).

The performance of our idealized device for finite-length sidewalls will be characterized by the *capture-width ratio* W defined as

$$W = P/P_{\mathrm{W}}. \tag{2.8}$$

All that is required is an estimate of the quantities Z and X in (2.4) in order to determine W from (2.4) and (2.8) for any particular value of A over a range of different frequencies. The quantities I and C appearing in (2.3) are determined from the geometry of the device, while the frequency-dependent hydrodynamic coefficients M, B and X are less easy to determine and in general require a considerable amount of numerical work.

2.2. *An approximate method*

Before embarking on a full numerical procedure for determining M, B and X, we shall derive a simple approximate method which will turn out to have wide validity. To clarify the notation we shall use the superscript h to denote quantities associated with the 'harbour' device comprising the rectangular block plus sidewalls of length L. Where the h is *not* displayed, the quantities refer to the case $L = \infty$ corresponding to the device positioned at one end of a semi-infinite wave tank.

In order to proceed further, it is assumed that $kL \gg 1$, where the incident wavelength λ satisfies $k = 2\pi/\lambda$. Under this assumption we can regard the active front face of the device bounded by the sidewalls of length L as responding to an *infinite* train of plane waves. Again the wavefield approaching the open end $x = L$ from $x < L$, $|y| \leqslant a$ can be regarded as a plane wave travelling down a semi-infinite waveguide bounded by two parallel rigid walls. It is known (see Noble 1958, p. 110) that such a wave is partly reflected back down the waveguide with a reflection coefficient R and partly radiated at the open end. For radiation into an infinite domain, or in this case the open sea, Noble shows that

$$R = \mathrm{e}^{-ka}\,\mathrm{e}^{2\mathrm{i}k(L+l)}, \tag{2.9}$$

where the 'added length' l is given by

$$\frac{l(ka)}{a} = \pi^{-1}\left(1-\gamma+\log\frac{2\pi}{ka}\right)-(ka)^{-1}\sum_{n=1}^{\infty}\left\{\sin^{-1}\frac{ka}{n\pi}-\frac{ka}{n\pi}\right\}, \tag{2.10}$$

where $\gamma = 0.5772...$ is Euler's constant. For radiation out into an infinite channel of width $b \geqslant a$, parallel to the waveguide, it can be shown using the Wiener–Hopf technique that

$$R = -(1-\mu)\,e^{2ik(L+l)}, \tag{2.11}$$

where $\mu = a/b$, $ka < \pi$, and in this case the 'added length' is given as

$$\frac{l(ka)}{a} = \pi^{-1}\{\log\mu^{-1}+(1-\mu)\,\mu^{-1}\log(1-\mu)^{-1}\} + \sum_{n=1}^{\infty}\left\{\sin^{-1}\frac{\mu^{-1}ka}{n\pi}-\sin^{-1}\frac{ka}{n\pi}-\sin^{-1}\frac{\mu^{-1}(1-\mu)\,ka}{n\pi}\right\}. \tag{2.12}$$

The simpler result $|R| = 1-\mu$ follows from a simple application of Green's theorem to the potential and to the functions $\exp(\pm ikx)$, which are also possible potentials in this case. This solution will be relevant to experimental work on 'harbour' devices in wave tanks where the influence of the sidewalls cannot be ignored.

Consider the exciting force X^{h} on the *fixed* front due to a plane wave of amplitude A travelling in a direction parallel to the sidewalls. The only effect of the (thin) sidewalls is to guide a slice of the incident wave towards the absorber, where it will be reflected as a plane wave of (complex) amplitude Ar, where r is the reflection coefficient for a fixed device. On reaching the open end it will be partly radiated out and partly reflected back towards the absorber as a wave of amplitude ArR. This process of multiple reflection will continue indefinitely, with the net effect that a wave of amplitude $A/(1-rR)$ will be incident on the front face. It follows that

$$X^{\mathrm{h}} = X/(1-rR). \tag{2.13}$$

Next, consider the applied force $Y_{\mathrm{e}}^{\mathrm{h}}$ necessary to maintain the front face at velocity U in the absence of the incident wave. We have

$$Y_{\mathrm{e}}^{\mathrm{h}} = Z^{\mathrm{h}}U, \tag{2.14}$$

whilst if $L = \infty$

$$Y_{\mathrm{e}} = ZU. \tag{2.15}$$

Now the effect of a velocity of U on the front face, under the assumption $kL \gg 1$, is initially to produce a wave of amplitude UC^{+} travelling away from the device. (Here UC^{+} is the (complex) wave amplitude radiated down a semi-infinite channel ($L = \infty$) owing to oscillations with (complex) velocity U). When this wave reaches the open end it will be partially reflected back down the wave guide, and after multiple reflections gives rise to an apparent incident wave of amplitude $UC^{+}R/(1-rR)$. Thus the *finite-L* case can be regarded as equivalent to the *infinite-L* case, provided that the applied force is augmented by a force that negates that produced by the apparent incident wave.

It follows that

$$Z^{\mathrm{h}}U = ZU-\frac{XUC^{+}R}{A(1-rR)}. \tag{2.16}$$

To simplify this it is necessary to use two relations relating X, C^+ and r:

$$X = \frac{-4P_W C^+}{A} \quad \text{(Haskind)} \tag{2.17}$$

and

$$r = \frac{C^+}{\overline{C^+}} \quad \text{(Newman)}. \tag{2.18}$$

As a result, using in addition the identity (2.7) and noting that $B = \frac{1}{2}(Z+\bar{Z})$, (2.16) reduces to

$$Z^{\mathrm{h}} = \frac{Z+rR\bar{Z}}{1-rR}. \tag{2.19}$$

In particular

$$\operatorname{Re} Z^{\mathrm{h}} = B^{\mathrm{h}} = \frac{B(1-|R|^2)}{|1-rR|^2}. \tag{2.20}$$

Substitution of (2.13) and (2.20) into (2.4) gives,

$$W^{\mathrm{h}} = \frac{1}{1-|R|^2}\left(1-\frac{|A-\bar{Z}^{\mathrm{h}}|^2}{|A+Z^{\mathrm{h}}|^2}\right), \tag{2.21}$$

showing that the maximum capture-width ratio of a device with projecting sidewalls is just

$$W^{\mathrm{h}}_{\max} = (1-|R|^2)^{-1}. \tag{2.22}$$

This gives

$$W^{\mathrm{h}}_{\max} = (1-\mathrm{e}^{-2ka})^{-1} \tag{2.23}$$

for a device in the open sea. Use of (2.21) in addition gives, after rearrangement,

$$W^{\mathrm{h}} = \frac{|A+Z|^2-|A-\bar{Z}|^2}{|(A+Z)-rR(A-\bar{Z})|^2}, \tag{2.24}$$

an expression for the capture-width ratio entirely in terms of the impedance Z for the effectively two-dimensional problem with $L = \infty$, the complex reflection coefficients r, R, and the power take-off parameter A.

The modification to W^{h} required for obliquely incident waves making an angle θ with the sidewalls is straightforward and only affects X^{h} and not Z^{h}. Thus the obliquely incident wave of amplitude A will be guided by the sidewalls towards the absorbing front face as a wave of amplitude A' dependent on θ before multiple reflections take place. Thus (2.13) needs to be modified by the term A'/A on the right-hand side, whilst (2.21) and (2.24) require the multiplication factor $|A'(\theta)/A|^2$ on the right-hand side. This factor has a particularly simple expression given in Noble (1958, equation (3.26)), so that, for example, from (2.22) with $|R| = \exp(-ka)$,

$$W^{\mathrm{h}}_{\max}(\theta) = \mathrm{e}^{-ka(1-\cos\theta)}\,(ka\sin\theta)^{-1}\sin(ka\sin\theta)\,W^{\mathrm{h}}_{\max}(0), \tag{2.25}$$

in an obvious notation.

Curves of (2.23) and (2.25) showing the variation of $W^{\mathrm{h}}_{\max}$ with ka for different angles of incidence are presented in figure 2. Also shown for comparison are curves of $W_{\max}$ for an isolated absorbing device like a buoy which oscillates about a vertical axis of symmetry. For vertical (heave) oscillations $W_{\max} = (2ka)^{-1}$, whilst for horizontal (sway or surge) oscillations $W_{\max} = (ka)^{-1}\cos^2\theta$. Notice that

$$W^{\mathrm{h}}_{\max}(0) \to (2ka)^{-1} \quad \text{as} \quad ka \to 0,$$

as might be anticipated on physical grounds since the device appears to the waves like an isolated pulsating source.

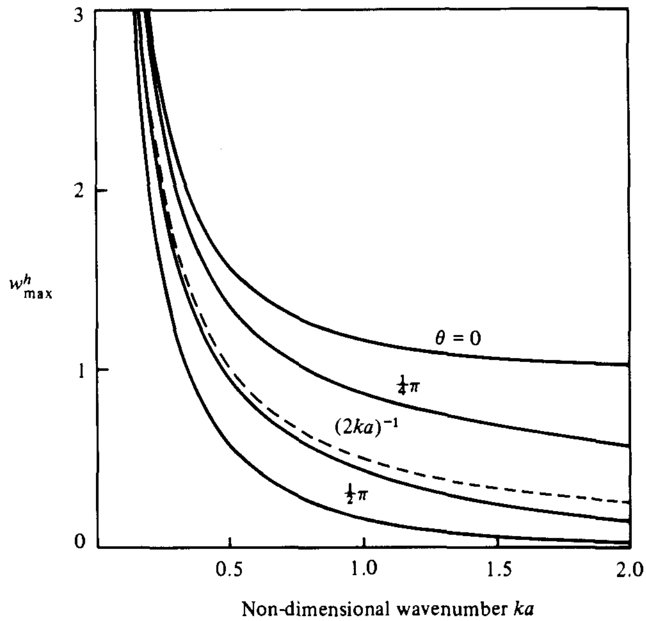

FIGURE 2. The maximum capture-width ratio for a harbour system for various angles of incidence in comparison with the result $(2ka)^{-1}$ for a heaving axisymmetric structure.

3. A numerical solution

The most direct way of solving for the hydrodynamic properties of the 'harbour' system comprising a rectangular block with oscillating front face, plus projecting sidewalls, would be simply to use a source-distribution method common in ship-hydrodynamics theory. In this method wave sources are distributed over the entire rigid body and the known normal velocity of the body provides an integral equation for the unknown source density. It is usual to discretize this integral equation and solve the resulting system of algebraic equations.

In the present problem such an approach would not work, since a source distribution is totally inappropriate for modelling the different flows either side of the thin projecting walls shown in figure 1. If a modification of this method were to be used, a dipole distribution would be more appropriate physically. Instead we choose an alternative method, which involves solving for the flow *inside* the 'harbour' and matching this with another solution valid *outside*, the two solutions being matched across the connecting region, the harbour mouth. This approach also has the advantage of allowing for the predominantly two-dimensional nature of the solution within the harbour and matching with a fully three-dimensional solution outside, instead of attempting to use a representation valid in both regions.

3.1. *The outer solution*

The outer region is defined, by reference to figure 1, to be the fluid region exterior to the rectangular block, closed by the plane S_1: $x = L$, $0 \leqslant z \leqslant h$, $|y| \leqslant a$. We define $S_B = S_1 \cup S_2$, with S_2 the fixed rigid boundary $-d \leqslant x \leqslant L$, $0 \leqslant z \leqslant h$, $y = \pm a$, and $x = -d$, $0 \leqslant z \leqslant h$, $|y| \leqslant a$. Let $\phi_R(\boldsymbol{r})$ ($\phi_S(\boldsymbol{r})$) denote the time-independent radiation (scattering) potential in the outer region and let $\phi_0(\boldsymbol{r})$ denote a given incident-wave potential. Further, let $G(\boldsymbol{r}, \boldsymbol{r}')$ denote the three-dimensional finite-depth source potential given by, for example, Wehausen & Laitone (1960). An application of Green's theorem in the usual manner gives the integral relation

$$2\pi\phi_{\mathrm{R}}(\boldsymbol{r}) = \int_{S_{\mathrm{B}}} \left\{ \frac{\partial G}{\partial n}(\boldsymbol{r}, \boldsymbol{r}')\, \phi_{\mathrm{R}}(\boldsymbol{r}') - G(\boldsymbol{r}, \boldsymbol{r}') \frac{\partial \phi_{\mathrm{R}}}{\partial n}(\boldsymbol{r}') \right\} \mathrm{d}S. \tag{3.1}$$

Here $\boldsymbol{r}$ lies on S_{B}, and the point $\boldsymbol{r} = \boldsymbol{r}'$ is excluded from the integration. This result assumes that the usual linearized equation of water-wave theory are satisfied by ϕ_{R}, and also that ϕ_{R}, like G, behaves like an outgoing wave at large distances. The scattering problem can also be treated in this fashion, since (3.1) is also valid with ϕ_{R} replaced by the difference potential

$$\phi_{\mathrm{D}} = \phi_{\mathrm{S}} - \phi_0, \tag{3.2}$$

which is also outgoing at large distances. Now for each problem

$$\frac{\partial \phi_{\mathrm{R,S}}}{\partial n} = \begin{cases} V(y,z) & \text{on } S_1, \\ 0 & \text{on } S_2 \end{cases}$$

for some function $V(y,z)$, and (3.1) reduces to an integral equation for ϕ_{D}, namely

$$2\pi\phi_{\mathrm{D}}(\boldsymbol{r}) = \int_{S_{\mathrm{B}}} \frac{\partial G}{\partial n}(\boldsymbol{r}, \boldsymbol{r}')\, \phi_{\mathrm{D}}(\boldsymbol{r}')\, \mathrm{d}S - \int_{S_1} G(\boldsymbol{r}, \boldsymbol{r}')\, V(\boldsymbol{r}')\, \mathrm{d}S + \int_{S_{\mathrm{B}}} G(\boldsymbol{r}, \boldsymbol{r}') \frac{\partial \phi_0}{\partial n}(\boldsymbol{r}')\, \mathrm{d}S, \tag{3.3}$$

or, for ϕ_{S}, by using (3.2).

The corresponding integral equation for ϕ_{R} is obtained from (3.3) by replacing ϕ_{D} by ϕ_{R} and dropping the last term.

3.2. *The inner solution*

We seek a harmonic function ψ in the region $0 \leqslant x \leqslant L$, $|y| \leqslant a$, $0 \leqslant z \leqslant h$, satisfying the conditions

$$\omega^2 \psi + g \frac{\partial \psi}{\partial z} = 0 \quad (z = 0,\ 0 \leqslant x \leqslant L,\ |y| \leqslant a), \tag{3.4}$$

$$\frac{\partial \psi}{\partial y} = 0 \quad (y = \pm a,\ 0 \leqslant x \leqslant L,\ 0 \leqslant z \leqslant h), \tag{3.5}$$

$$\frac{\partial \psi}{\partial z} = 0 \quad (z = +h,\ 0 \leqslant x \leqslant L,\ |y| \leqslant a), \tag{3.6}$$

whilst on S_0: $x = 0$, $|y| \leqslant a$, $0 \leqslant z \leqslant h$ we shall assume ψ is normalized so that

$$\frac{\partial \psi}{\partial x} = U = \begin{cases} 1 & \text{(radiation problem)}, \\ 0 & \text{(scattering problem)}. \end{cases} \tag{3.7}$$

Now separation of variables shows that the general solution may be expressed in the form

$$\psi(x,y,z) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} (A_{nm}\, \mathrm{e}^{-\alpha_{nm} x} + B_{nm}\, \mathrm{e}^{\alpha_{nm} x})\, \psi_{nm}(y,z), \tag{3.8}$$

where

$$\psi_{nm}(y,z) = N_n \cos \mu_n (h - z) \cos \frac{m\pi y}{a},$$

with
$$N_n^{-2} = (2\mu_n h + \sin 2\mu_n h)\frac{a}{4\mu_n},$$

and
$$\alpha_{nm} = \left(\mu_n^2 + \frac{m^2\pi^2}{a^2}\right)^{\frac{1}{2}}.$$

Here $\mu_0 = -\mathrm{i}k$, and the $\mu_n\,(n \geqslant 1)$ are the positive real roots of

$$\omega^2 g^{-1} + \mu \tan \mu h = 0,$$

whilst k is the positive real root of

$$\omega^2 g^{-1} = k \tanh kh.$$

Since $ka < n\pi$, $\alpha_{n0} > 0$, whilst $\alpha_{00} = -\mathrm{i}k$.

The set $\{\psi_{nm}(y,z)\}$, $n, m = 0, 1, \ldots$, can be shown to be orthonormal over $0 \leqslant z \leqslant h$, $|y| \leqslant a$.

Conditions (3.4)–(3.6) are automatically satisfied by the choice of solution, whilst (3.7) is satisfied if

$$\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \alpha_{nm}(B_{nm} - A_{nm})\,\psi_{nm}(y,z) = U.$$

It follows that

$$\alpha_{nm}(B_{nm} - A_{nm}) = U_{nm} \equiv \int_0^h \mathrm{d}z \int_{-a}^{a} U\psi_{nm}(y,z)\,\mathrm{d}y. \qquad (3.9)$$

Now continuity of pressure across S_1 gives

$$B_{nm}\,\mathrm{e}^{\alpha_{nm}L} + A_{nm}\,\mathrm{e}^{-\alpha_{nm}L} = \int_0^h \mathrm{d}z \int_{-a}^{a} \phi(y,z)\,\psi_{nm}(y,z)\,\mathrm{d}y, \qquad (3.10)$$

where $\phi(y,z)$ is the potential for either the scattering or radiation problem in the outer region. Equations (3.9) and (3.10) permit all constants to be determined in terms of known quantities or integrals of ϕ over S_1.

On S_1 continuity of horizontal velocity gives

$$\frac{\partial\psi}{\partial x}(L,y,z) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \alpha_{nm}(B_{nm}\,\mathrm{e}^{\alpha_{nm}L} - A_{nm}\,\mathrm{e}^{-\alpha_{nm}L})\,\psi_{nm}(y,z) = V(y,z),$$

and substitution of this into (3.3) gives an integral equation for ϕ_{R} or ϕ_{S} over $S_{\mathrm{B}} = S_1 \cup S_2$, after using (3.2).

The numerical solution of (3.3) proceeds as follows. The surface $S_1 \cup S_2$ is partitioned into N plane-area facets, a typical one having centroid $\boldsymbol{x}_i$ and area ΔS_i, and the unknown potential is assumed to be constant over each facet. This reduces the integral equation to a system of algebraic equations for the values of $\phi_{\mathrm{R,S}}$ at the centroid of each facet. Care must be taken to avoid the condition $2kL = n\pi$, with n an integer, since this corresponds to resonance in the inner region. Further details of the numerical procedure can be found in Count (1983).

Once $\phi_{\mathrm{R,S}}$ are determined on S_1, all constants are fully determined and (3.8) can be used to determine the force on S_0 from which Z^{h} and X^{h} follow.

4. Results

In order to compare the values of Z^{h} and X^{h} determined from the full numerical procedure just described, and the approximate expressions given by (2.19) and (2.13), it is necessary to determine Z and X for the device in a semi-infinite channel. The

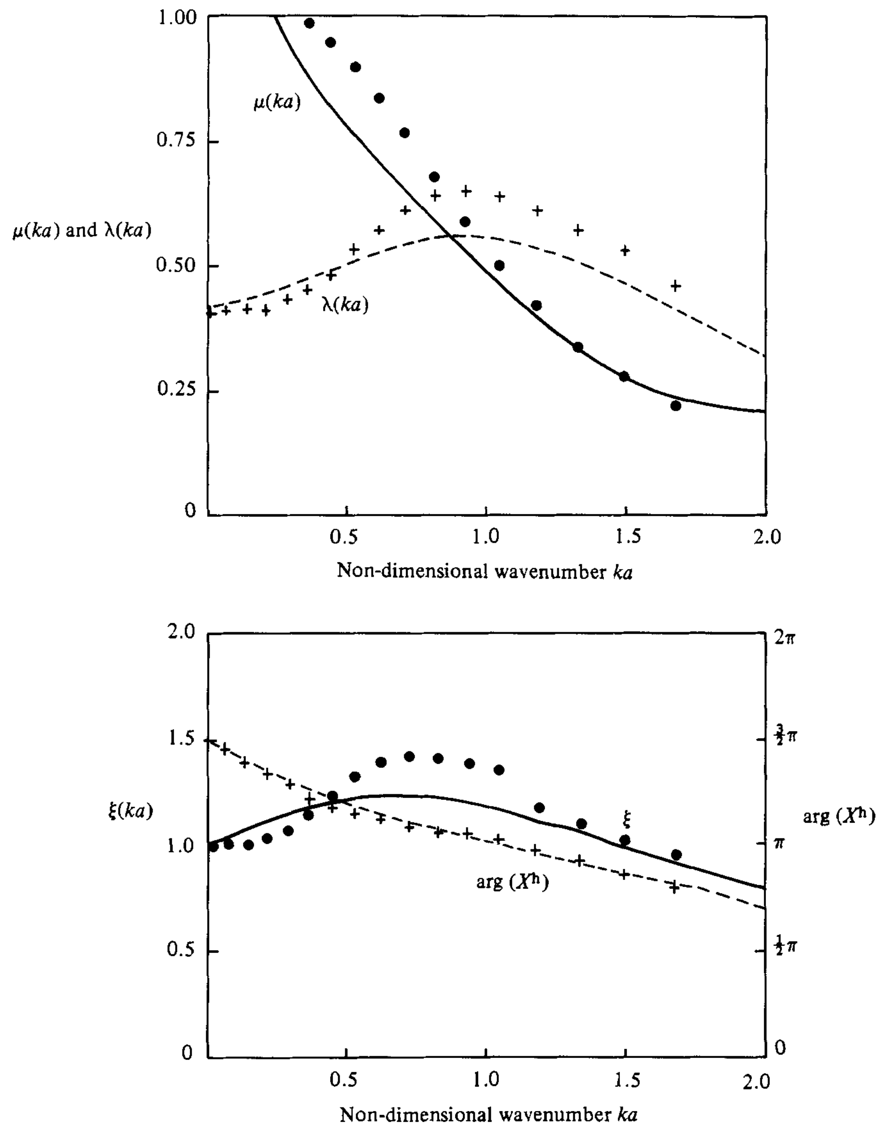

FIGURE 3. Comparison between the approximate theory (solid and dashed lines) and numerical calculations (●, +) for $L/d = 0$.

scattering problem is extremely simple, since the solution is a standing wave of amplitude $2A$ corresponding to the complete reflection of the incident wave at the *fixed* front face. The solution to the radiation problem is just a special case of the Havelock's (1929) wavemaker theory, and can be determined from (3.8) and (3.9) by putting $m = 0$, $B_{nm} = 0$. Thus for this particular case the solution is given explicitly from (3.8) with A_{mn} given by (3.9), and Z and X can be determined directly. With $r = 1$ for the simple idealized device considered here, and R given by (2.9) and (2.10), the expressions (2.13) and (2.19) can now be computed and compared with the expressions for X^{h} and Z^{h} obtained from the full numerical approach.

It is convenient, by reference to figure 1, to non-dimensionalize in the following manner. The mass of the active front face of the block is taken to be $I = 2adh\rho$, which

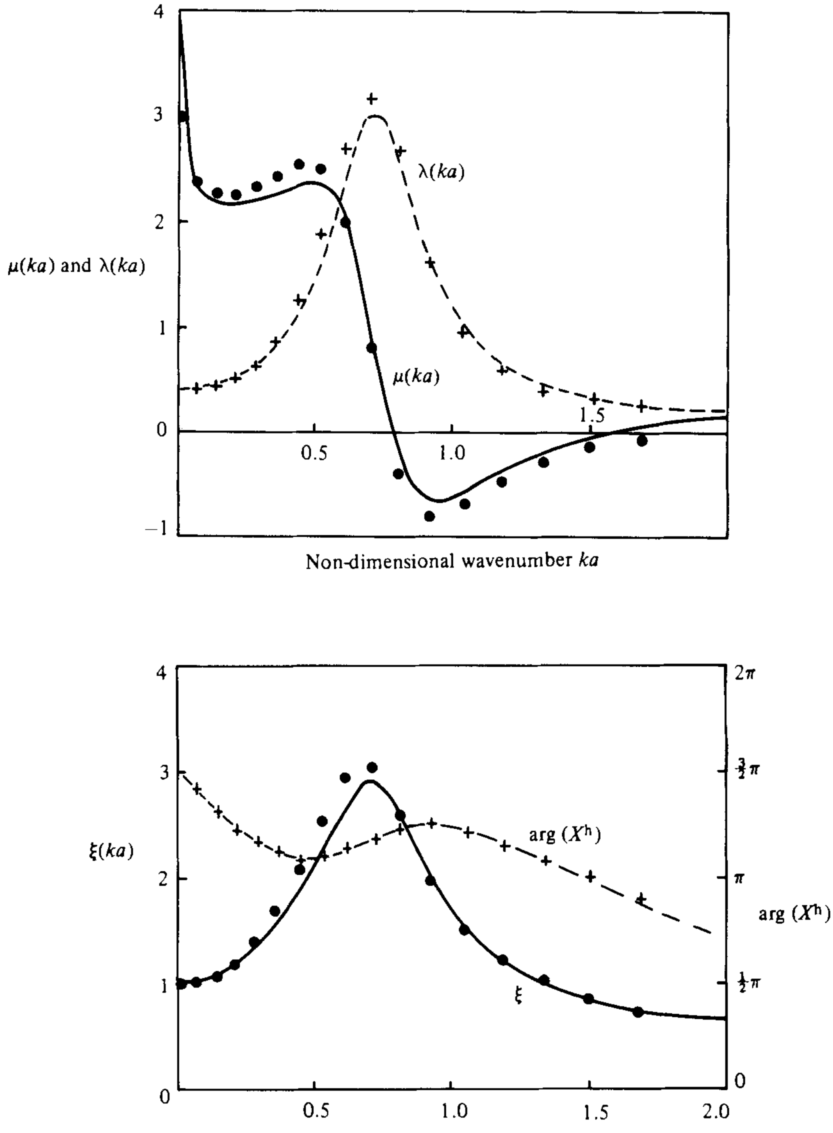

FIGURE 4. Comparison between the approximate theory (solid and dashed lines) and numerical calculations (●, +) for $L/d = 1$.

is the equivalent mass of water behind the front face. Then the frequency-dependent added-mass and damping coefficients M and B are non-dimensionalized by writing

$$\mu = \frac{M^{\mathrm{h}}}{I}, \quad \lambda = \frac{B^{\mathrm{h}}}{I\omega},$$

whilst the exciting force X^{h} is written

$$\xi = |X^{\mathrm{h}}|/2a\rho g h A,$$

being non-dimensionalized using the increase in hydrostatic force on the front face due to an increase A in water elevation. This ensures that $\xi = 1$ at zero frequency.

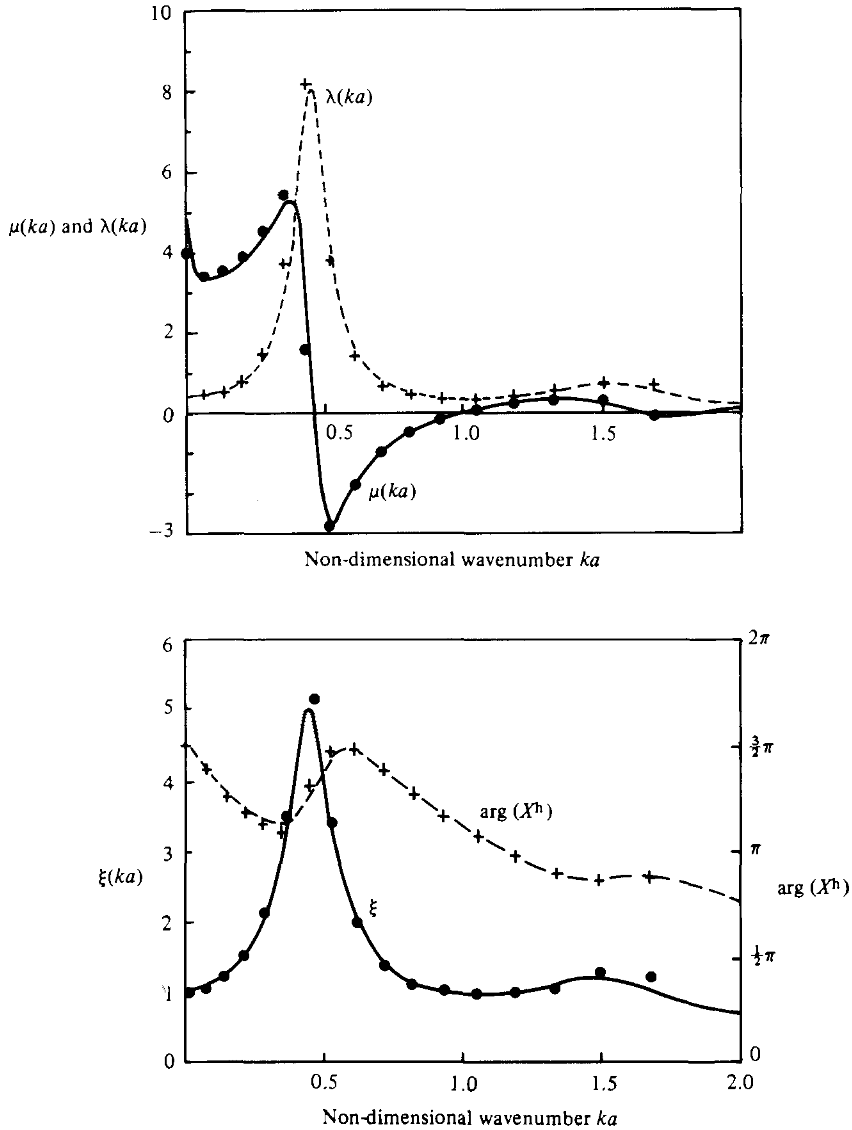

FIGURE 5. Comparison between the approximate theory (solid and dashed lines) and numerical calculations (●, +) for $L/d = 2$.

The variation of μ, λ, ξ and $\arg X^{\mathrm{h}}$ with dimensionless wavenumber ka for different lengths of sidewalls characterized by L/d are shown in figures 3–5 using both the full numerical results and the approximate theory. The dimensions $d = 1.2$ m, $h = 1.3$ m and $a = 1$ m correspond to typical model-sized devices for wave-tank testing purposes and provide values of the parameters h/d and a/d for use in the calculations.

It is clear from figure 5 that for $L/d = 2$ the approximate theory is in excellent agreement with the numerical results over the entire frequency range of interest. For $L/d = 1$ the agreement is less good, as figure 4 shows, although the essential features of, for example, the added-mass coefficient $\mu(ka)$ are preserved. It is interesting to note that the effect of increasing the length of the harbour walls is to produce a narrower, more peaked damping coefficient $\lambda(ka)$ as a function of frequency, while

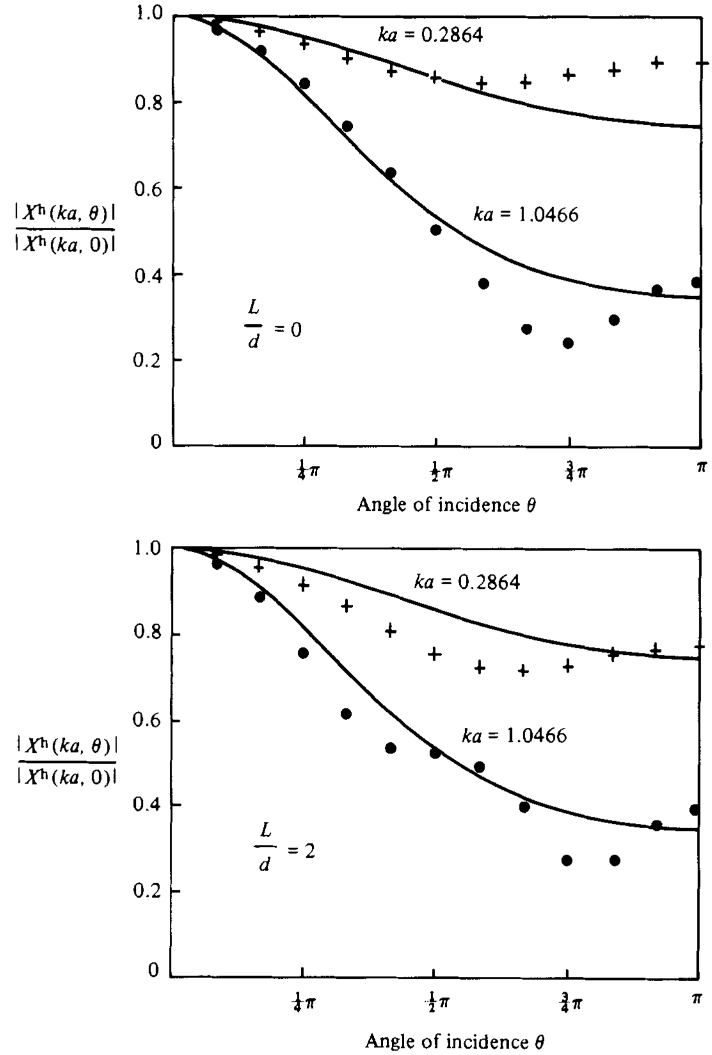

FIGURE 6. Comparison of the angular dependence of the exciting-force amplitude using the approximate theory (solid lines) and numerical calculations (●, +) for $ka = 0.2864$, 1.0466 and $L/d = 0, 2$.

the added-mass coefficient $\mu(ka)$ displays more rapid variations and an increasing number of zeros as L/d increases. The case $L/d = 0$ corresponds to the absence of projecting sidewalls. The numerical solution in this case was determined using the matching technique described here, and also, as a check, a direct boundary-element source-distribution method was used since now there is no problem over the thin sidewalls. The results for this case are shown in figure 3, and it is of interest to compare them with the approximate method. Despite the fact that there are *no* sidewalls, the approximate method still yields results with $L/d = 0$, and in fact is still in fair agreement with the numerical results.

In addition, results for X^{h} have been calculated for obliquely incident waves. The analytic expression for the angular behaviour was given by (2.25), where

$$\left|\frac{X^{\mathrm{h}}(ka,\theta)}{X^{\mathrm{h}}(ka,0)}\right|^2 = \mathrm{e}^{-ka(1-\cos\theta)}\,(ka\,\sin\theta)^{-1}\sin(ka\,\sin\theta),$$

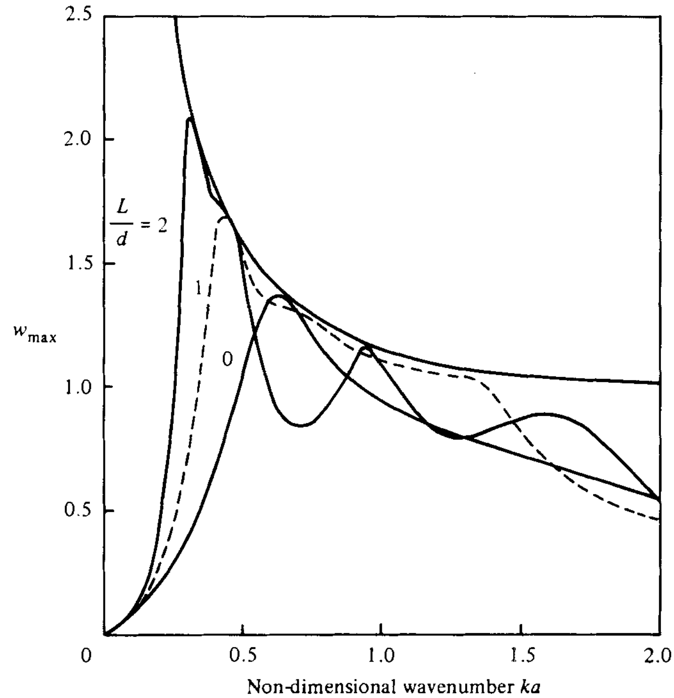

FIGURE 7. The capture-width ratio for $L/d = 0$, 1, 2 using an optimized (real) damping constant Λ compared with W^{h}_{max} for normally incident waves ($\theta = 0$).

in an obvious notation. In figure 6 $|X^{h}(ka, \theta)|/X^{h}(ka, 0)|$ is shown in order to compare the simple expression given above with the numerically computed values for two values of the ka and $L/d = 0$, 2. In this case the correlation is not so impressive, although the general shapes of the curves are the same.

The differences between the two methods can be explained by the influence of the wave scattering from the outside walls of the device. For the case $\theta = 0$ we would expect the scattered wave to have little influence, since the sidewalls are aligned with the propagation direction, whereas for $\theta > 0$ there will be increasingly important scattering from the side and rear walls of the device. Moreover, the finite nature of the system will be important in determining these scattering effects. For example, if $\theta = \pi$, the theory based on the assumption $L/d \gg 1$ assumes that a plane incident wave is travelling along the sidewalls toward the harbour mouth, whereas for finite values of L/d the influence of the rear wall will not have decayed sufficiently for this assumption to be valid.

Overall, the approximations given by (2.13) and (2.19) can be expected to remain good over a much wider range than expected, particularly for $\theta = 0$. Therefore these formulae may be used with confidence for parametric studies on harbour systems with more realistic devices than the simple configuration used for this study.

Having confirmed the applicability of the approximate formulae, it remains to assess the effect on performance of our device of adding sidewalls of varying length. To do this a choice of $C \neq 0$ must be made in (2.3) so that the device has a natural resonance, as would be the case for a realistic device. We choose $C = 2a\rho g d = Ig/h$ as being typical, and from (2.3) this provides a first resonant frequency at the lowest ω satisfying

$$\omega^2 h/g = (1+\mu)^{-1}. \tag{4.1}$$

The assumed linear power take-off will be characterized by the real positive constant Λ. From (2.4) maximum efficiency is achieved when $\Lambda = |Z^h|$, and this value was chosen at each frequency computed. The limit $(1-e^{-2ka})^{-1}$ can only be achieved if $\Lambda = Z^h$, which for *real* Λ requires $\operatorname{Im} Z^h = 0$ or (4.1) to be satisfied.

We have, for $\Lambda = |Z^h|$,

$$W_{max} = \frac{(1-e^{-2ka})^{-1} 2B^h}{(|Z^h|+B^h)},$$

and this expression is sketched in figure 7 as a function of ka for different values of L/d. Also shown is the upper limit on the capture-width ratio predicted by the approximate theory and given by (2.23).

It is clear that the addition of sidewalls produces a number of effects on the performance of the device. First, the peak performance is increased and also shifted to the lower end of the frequency range. Secondly, the curves for $L/d = 1, 2$ contain more than one peak that approach the upper limit of performance. Both of these effects can be explained in terms of the modifications to λ and μ as L/d increases. Thus the rapid changes in μ shown in figures 4 and 5 for $L/d = 1, 2$ produce extra solutions to the equation (4.1) for the resonant frequencies. The peakiness of the curve of $\lambda(ka)$ near the first zero of $\mu(ka)$ results in a similar narrowing in the performance curves near the maximum values.

It is of interest to note that the curve for $L/d = 1$ is close to the upper limit of performance over a wide frequency range. This has been achieved using a simple linear resistive damping control plus harbour walls, thus avoiding the need for sophisticated control mechanisms involving complex values of Λ.

5. Conclusion

A theoretical model has been developed for the hydrodynamic performance of a wave-energy device equipped with projecting sidewalls. A simple approximate theory enabled predictions to be made of its hydrodynamical characteristics of added mass, damping and exciting force in terms of the characteristics of a simple two-dimensional mode. Good agreement was obtained between the approximate results and the results based on a full numerical treatment. The approximation technique may also be used to predict the efficiency of the device plus sidewalls when operating in the middle of a wave tank band of width $b \geqslant a$, which will be more akin to many experimental configurations.

In this paper a simple device was chosen for analytical convenience. The performance results indicate that substantial changes occur on adding a harbour, and by design these can be extremely beneficial. There is no reason to believe that the same effect would not be detected for any other wave-energy device that operates well in a semi-infinite channel, including the oscillating water column device being considered at the National Engineering Laboratory (Moody & Elliot 1982).

As a result, the analyses presented in this paper have confirmed the claims of Ambli *et al.* (1982) that the addition of projecting sidewalls will increase the energy capture of a wave-energy device. This opens up the possibility that with careful design it may be possible to substantially improve the economic potential of wave-energy systems, and it is to be hoped that this paper has demonstrated that this concept is exciting and merits futher investigation.

This work is the result of a continuing liaison between the authors in the wave-energy field. Results for the numerical model are published by permission of the CEGB and describe work undertaken at Marchwood Engineering Laboratories. Development of the approximate model forms part of a continuing research programme in the Department of Mathematics, University of Bristol, supported by the SERC Marine Technology Directorate under Grant GR/B/76720, and initially by NTNF, Norway, while D.V.E. was a visitor at the University of Trondheim in 1982.

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